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REMARK. On again looking up our reference we find that we quoted the reference correctly, though as Mr. Heal observes, it seems that our reference is wrong. The value of the series

$$1 - \frac{1}{3^{2n+1}} + \frac{1}{5^{2n+1}} - \frac{1}{7^{2n+1}} + \text{etc.} = \frac{\pi^3}{32}$$

is also given in Carr's *Synopsis of Pure Mathematics*, page 432. Since it now appears that the *value* of the integral is not known in finite terms, we will renew our offer to give the person finding such value a year's subscription to the MONTHLY. ED.

125. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

Show that the *complete primitive* of the differential equation

$$\left[ \tan^{-1}(x) - \frac{x}{1+x^2} \right] \frac{dy^2}{dx^2} = 1 \left[ \frac{x}{(1+x^2)^2} \right] \left[ x \frac{dy}{dx} - y \right]$$

is  $y = C \tan^{-1}(x) + cx$ .

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

$w$  being one solution of

$$\frac{d^2y}{dx^2} + P \frac{dy}{dx} + Qy = 0 \dots (1),$$

and  $P, Q$  functions of  $x, y = vw \dots (2)$  is given, by the usual theory, from

$$\frac{d^2v}{dx^2} + \left( \frac{2}{w} \frac{dw}{dx} + P \right) \frac{dv}{dx} = 0 \dots (3), \text{ or } v = c_2 + c_1 \int \frac{dx}{w^2} e^{-\int P dx} \dots (4).$$

In the given equation,  $w = x \dots (5)$ , and

$$P = -\frac{2x^2}{(1+x^2)^2} \div \left[ \tan^{-1}x - \frac{x}{1+x^2} \right] \dots (6).$$

(4) now easily becomes

$$\begin{aligned} v &= c_2 + c_1 \int \frac{dx}{x^2} \left( \tan^{-1}x - \frac{x}{1+x^2} \right) = c_2 + c_1 \left( \int \frac{\tan^{-1}x}{x^2} - \int \frac{dx}{x(1+x^2)} \right) \\ &= c_2 - c_1 \frac{\tan^{-1}x}{x^2} \dots (7), \end{aligned}$$

and (2) is  $y = c_2x - c_1 \tan^{-1}x \dots (8)$ .

II. Solution by W. W. BEMAN, A. M., Professor of Mathematics, State University, Ann Arbor, Mich.

$$\left(\tan^{-1}x - \frac{x}{1+x^2}\right) \frac{d^2y}{dx^2} = \frac{2x}{(1+x^2)^2} \left(x \frac{dy}{dx} - y\right).$$

Writing the equation in the form

$$\frac{d}{dx} \left( \frac{\tan^{-1}x}{x} \right) \cdot \frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{1}{1+x^2} \right) \cdot \frac{d}{dx} \left( \frac{y}{x} \right)$$

it is obvious that  $y = \tan^{-1}x$  and  $y = x$  are independent particular integrals. Hence the complete primitive is  $y = c_1 \tan^{-1}x + c_2 x$ .

Solutions of this problem were also received from L. C. WALKER, and G. B. M. ZERR.

126. Proposed by JOHN M. COLAW, A. M., Monterey, Va.

Find the volume contained between the conical surface whose equation is  $z = a - \sqrt{x^2 + y^2}$ , and the planes whose equations are  $x = z$  and  $x = 0$  by the formula  $\iiint dx dy dz$ . [Todhunter's *Integral Calculus*.]

Solution by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, Gloucestershire, England.

The cone clearly extends from vertex  $(0, 0, a)$  towards  $z = 0$ . Hence in

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} \int_{z_1}^{z_2} dx dy dz$$

we have  $z_1 = x$ ;  $z_2 = a - \sqrt{x^2 + y^2}$ ;  $x = a - \sqrt{x^2 + y^2}$ .

$\therefore y^2 = a^2 - 2ax$ ,  $x_1 = 0$ , and for  $x_2$  we have  $y_1 = y_2$ .  $\therefore x = \frac{1}{2}a$ .

$$\begin{aligned} \therefore \text{Volume} &= \int_0^{\frac{1}{2}a} \int_{y_1}^{y_2} [a - x - \sqrt{x^2 + y^2}] dx dy \\ &= \int_0^{\frac{1}{2}a} \left[ (a-x)(y_2 - y_1) - \frac{y}{2} \sqrt{x^2 + y^2} - \frac{x^2}{2} \log y + \sqrt{x^2 + y^2} \right]_{y_1}^{y_2} dx \\ &= \int_0^{\frac{1}{2}a} \left[ 2(a-x) \sqrt{a^2 - 2ax} - \sqrt{a^2 - 2ax}(a-x) - \frac{x^2}{2} \log \frac{a-x + \sqrt{a^2 - 2ax}}{a-x - \sqrt{a^2 - 2ax}} \right] dx. \end{aligned}$$

Put  $2x = a \sin^2 \phi$ .

$$\begin{aligned} \therefore V &= \int_0^{\frac{1}{2}\pi} a \sin \phi \cos \phi d\phi \left[ (1 + \cos \phi) \frac{a^2}{2} \cos \phi - \frac{a^2}{8} \sin^4 \phi \log \left( \frac{1 + \cos \phi}{1 - \cos \phi} \right)^2 \right] \\ &= \frac{a^3}{2} \int_0^{\frac{1}{2}\pi} \left[ \sin \phi \cos^2 \phi + \sin \phi \cos^4 \phi - \sin^5 \phi \cos \phi \log \frac{1 + \cos \phi}{\sin \phi} \right] d\phi \end{aligned}$$